MA 242 Section 6 – Test 1 – Review

This review sheet contains questions that are similar to what I will ask on the test. The actual test will have fewer questions. Remember, the test will take place during the first 50 minutes of class, then we will have the usual 10 minute break, then a regular lecture for the remaining 50 minutes.

Vectors, lines, and planes:

(1) Compute the vectors OP, OQ, and PQ where $P = (1, 2, 3)$, $Q = (5, 3, -3)$ and $O = (0, 0, 0)$. Compute the distances between P and Q, P and O and O and Q.
(2) Find a unit vector in the direction of $\langle 3, -4, 1\rangle$.
(3) Find the distance from $P = (4, 1, 2)$ to the xy-plane and to the z-axis.
(4) Find the midpoint between $(1, 4, 2)$ and $(-2, 3, 1)$.
(5) Find the equation of the sphere if one of its diameters has endpoints $(-2, 3, 1)$ and $(-7, 7, 4)$.
(6) a 20 pound weight hangs from two wires. Both wires are connected to a horizontal beam. The angle the beam makes with one wire is $30°$, the angle the beam makes with the other is $60°$. Find the tension in each wire.
(7) Compute the angle between:
- $\langle -4, 2\rangle$ and $\langle 1, 1\rangle$
- $\langle 5, 2, 3\rangle$ and $\langle 4, 1, 2\rangle$.

(8) Compute the angles in the triangle that has vertices at $(0, 0, 0)$, $(3, -1, 4)$, and $(1, 2, 3)$.
(9) Compute the vector projection of $\vec{a}$ onto $\vec{b}$ in each of the following. Then, express $\vec{a}$ as the sum of a vector parallel to $\vec{b}$, and a vector perpendicular to $\vec{b}$.
- $\vec{a} = \langle 1, 4\rangle$ and $\vec{b} = \langle -2, 3\rangle$
- $\vec{a} = \langle 0, -2, 1\rangle$ and $\vec{b} = \langle 1, 3, 4\rangle$.

(10) Compute the cross product of $\langle 1, 0, 2\rangle$ and $\langle 3, 0, -1$ *by hand*. Check your answer on a computing device.
(11) Compute the work done by a force $\langle 1, 4, 2\rangle$ done on an object that moves $\langle 1, 1, 1\rangle$.
(12) If you pull a wagon 50m by holding the handle an angle of $30°$ above the horizontal and applying 40N of force then how much work do you do on the wagon?
(13) Find the cross product of the following pairs of vectors:
- $\langle 3, -1, 2\rangle$ and $\langle 1, 1, 1\rangle$
- $\langle 1, 2, 3\rangle$ and $\langle 2, 4, 6\rangle$
- $3\hat{i} + \hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$.

(14) Compute the area of the parallelogram that has three of its vertices at $(0, 0, 3)$, $(1, 2, 3)$ and $(1, 1, 1)$.
(15) Find parametric equations for the line that contains point $(2, 1, 3)$ that goes in direction $\langle 2, 4, 4\rangle$.
(16) Find parametric equations for the line containing points $(1, 2, 3)$ and $(-1, 3, 1)$. At what point does it intersect the zy-plane?

(17) For the lines given by the following equations, determine whether or not they intersect. If they do, find the point of intersection.
- $\langle 1,3,1\rangle + t\langle 2,2,1\rangle$ and $\langle 3,-1,5\rangle + s\langle 4,-2,5\rangle$
- $\langle 3,2,1\rangle + t\langle 5,-4,3\rangle$ and $\langle 4,4,3\rangle + s\langle -10,8,-6\rangle$.
- $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$.

(18) Find the equation for the plane that contains the point $(-3,2,0)$ and has normal vector $\langle 3,2,1\rangle$.

(19) Find the equation for the plane containing the points $(3,1,3)$, $(-2,3,6)$ and $(1,1,0)$.

(20) At what point does the line with $2x+1 = y = 2z-2$ intersect the plane with equation $2x-2y+z=4$?

(21) Compute the angle between the planes with equations $3x+2y=2$ and $5x-6y+z=-2$.

(22) Find equations for the line of intersection of the planes $x+y-z=3$ and $-2x+z=0$.

(23) Section 1.2.7 problem 14.

Curves:

(1) Section 2.2.5 problems 1,2,13

(2) Section 2.3.4 problems 3,5,6,9,10

(3) Section 2.4.3 1,15,16

(4) Let $\vec{r}(t) = \langle t, t^2\rangle$. Find the equation for the osculating circle at the point $(1,1)$.

Fun stuff: We did not directly discuss how to do the following two problems but you might enjoy figuring them out (don't worry, you won't need to be able to do them on the test).

(1) Find the distance from the point $(1,2,3)$ to the plane with equation $x+y+z=2$.

(2) Find the distance from the point $(1,1,1)$ to the line with parametric equations $x = 3+t$, $y = 1-t$, $z = t$ (hint: this is a minimization problem - you'll probably need to take a derivative).

(3) Let $\vec{r}(t)$ be a vector valued function such that $\|\vec{r}(t)\| = 1$ for all t. Show that $\vec{r}(t)\cdot\vec{r}'(t) = 0$ for all t.